Hall Ticket Number:

Code No. : 13701 VC

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. (CBCS) III-Semester Main Examinations, December-2017

Bridge Course: Fundamentals of Linear Algebra and Vector Calculus

(Common to Civil, EEE, ECE & Mech. Engg.)

Time: **3** hours Max. Marks: **50**

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Part-A (15 Marks)

1. Find $\frac{dy}{dx}$, If y^3-3a$x^2 + x^3 = 0$. [1]
2. Evaluate $\int e^x (1 + \cot x + cot^2 x)dx$. [1]
3. Find the Gradient of $2xy + z^2$ at (1,-1,3). [1]
4. Evaluate $\int_0^2 \int_0^x y\, dydx$ [1]
5. If $\bar{n}$ is the unit outward drawn normal to any closed surface then $\int_v div\, \bar{n}\, dv$ [1]
6. Find First order partial derivatives of the function $\tan^{-1}(x + y)$ [2]
7. Evaluate $\int x^2 \log x\ dx$ [2]
8. If $\bar{f}$=$(x + 3y)\bar{\imath}$+$(y + 2z)\bar{\jmath}$+$(x + pz)\bar{k}$ *is solenoidal , find p.* [2]
9. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz\ dzdydx$ [2]
10. State Gauss's divergence theorem in a plane. [2]

Part-B (5 × 7 = 35 Marks)
(All bits carry equal marks)

11. a) If $z = f(x+ct) + g(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$.

    b) If $u = f(y - z, z - x, x - y)$ Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

12. a) Evaluate $\int \frac{\cos x}{1+sin^2 x} dx$.

    b) Evaluate $\int \frac{\sin x \cos x}{a^2 cos^2 x + b^2 sin^2 x} dx$.

13. a) Find the Directional derivative of $\emptyset = x^2yz + 4xz^2$ at (1,-2,-1) in the direction of 2i-j-2k.

    b) If $\bar{f}$ =grad$(x^3 + y^3 + z^3 - 3xyz)$ find div $\bar{f}$ *and* $curl\bar{f}$.

14. a) If $\bar{F}$ =3$xy\bar{\imath}$ -$y^2\bar{\jmath}$, find the work done by the force along the Curve $y = 2x^2$ in the xy −plane from (0,0) to (1,2).

    b) Evaluate $\int_s \bar{F} . \bar{n}\ ds$ where $\bar{F} = z\bar{\imath} + x\bar{\jmath} - 3y^2z\bar{k}$ and S is the Surface $x^2 + y^2 = 16$ included in the first octant between z=0 and z=5.

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15. a) Using Gauss divergence theorem, Show that

$\int_s (ax\bar{\imath} + by\bar{\jmath} + cz\bar{k})\ \bar{n}\, ds = \frac{4\pi}{3}(a+b+c)$, S: surface of the Sphere $x^2 + y^2 + z^2 = 1$.

b) Evaluate by Green's theorem $\int_c (x^2 - \cos hy)dx + (y + \sin x)d\,y$ Where C is the rectangle with vertices (0,0),$(\pi, 0)$, $(\pi, 1)$, (0,1)

16. a) If $f = x^3 + y^3 - 3axy$ Verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

b) Evaluate $\int sin^4 x\, dx$.

Answer any *two* of the following:

17. a) Find the angle between the surfaces $x^2 + xy + y^2z + 3xyz = 4$ *at* (1,2,1) (-1,1,-1)

b) If $\bar{F} = (2x^2 - 3z)\bar{\imath} - 2xy\bar{\jmath} - 4x\bar{k}$ then evaluate $\int_v div\ \bar{F}\, dv$, where v is the closed region bounded by the planes $x = 0, y = 0, z = 0$ *and* $2x + 2y + z = 4$.

c) Using Stoke's theorem

Evaluate $\int_c (x + y)dx + (2x - z)dy + (y + z)dz$ here c is the boundary of the triangle with vertices (2,0,0),(0,3,0)and (0,0,6).

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